Intersubjective Probability and Consensus Aggregation: Review and Bibliography

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Motivation for Intersubjective Probability

- The frequency interpretation of probability (along with repeated sampling) reflects an **operational** notion of objectivity.
- From an **operational** standpoint, the frequency theory, according to Genest and Zidek (1986), is sometimes "stretched to the limits of plausibility."
- Consider, for example, testing factors associated with the failure of a nuclear powerplant or a hydoelectric dam. These differ from operationally testing a toss of a die.
- The Ramsey-DeFinetti interpretation (sometimes called the "Bayesian interpretation") is treated in statistics literature as an alternative which is immune to this failure.
- According to Genest and Zidek:

> The Bayesian program would seem to be entirely satisfactory as a normative theory for the individual. However, groups of individuals are left stranded; no concept equivalent to the classical notion of objectivity is available to them.

- In the statistics literature, ideas of aggregation are developed independently in Weerahandi and Zidek (1981, 1983) and Dawid (1982). The latter calls this idea "intersubjectivity."
- According to these analyses, the results of an experiment would be called "objective" or "intersubjective" if the same concluson would be reached by a number of individuals faced with the same evidence. This is the **epistemic notion** of intersubjective probabilities.
- There are more practical reasons to study intersubjective probability; Savage (1954) calls this the **jury problem**: summarizing multiple judgments "at the end of the day".

Three Lines of Development

- There are three lines of literature deriving from three different fields focusing on this topic: the Statistical line, the Dutch Book line, and the Normative-Iterative line.
- The **statistical line** starts with Weerahandi and Zidek (1981) and Dawid (1982); these mostly focus on various mathematical methods for "averaging" forecasts.
- The **Dutch Book** line begins with Gillies (1992) and Ryder (1982), which extend Dutch Book arguments to social groups. Gillies argues that if groups have a "common interest" and access to intersubjective information flow, they must arrive at a single betting quotient for a given event. If they do not, the group is **incoherent** since a Dutch book can made against the group.

- The **Normative-Iterative** line begins with Leher and Wagner (1981), *Rational Consensus in Science and Society*. The authors propose an interative process in which individuals within a group are asked to assess "weights of authority" for all the members of a given group (including themselves). After a forecast is given by each member for a particular event space, the forecast of each member is transformed by a weight matrix. Lehrer and Wagner claim that the resulting probability vector for an individual is what they *ought* to believe given their assessment of "weights of authority."

- There are a number of questions of philosophical interest as a result of this wide literature.
  1. Does the existence of consensual probability function, $f : [0,1]^n \mapsto [0,1]$, for a given event make it more *objective* than any individual probability in its domain?
  2. Is the project of aggregation coherent, given that individual subjective probability distributions are frequently incoherent?
  3. If a social welfare function can be constructed for a group, can that measure group partial-belief?

Skepticism about the Project

Probabilities and Belief Probabilities do not adequately form an idiom for belief. (Baird, 1985)

Linguistic Skepticism "P(E) = $\frac{1}{2}$" is not equivalent in meaning for everyone.

Elicitation How are probabilities to be elicited?

Interpersonal Comparability Are probabilities interpersonally comparable? Is the scale of partial belief the same for everyone?

- Predd belongs to the statistical line of literature and therefore focuses upon methods of aggregation.
- Predd starts with a problem about **linear averaging**. Consider the following table

	Alice	Bob	Chris	Aggregate
X	**0.75**	0.50	1.00	**0.75**
Y	0.25	0.25	0.25	0.25
X $\vee$ Y	**0.25**	0.75	1.00	**0.67**

How to fix this problem?

- One suggestion is for the analyst to try to instill coherent probabilities through elicitation techniques.
- However, this does not work for abstention where the expert does not feel confident to provide an assessment of, say, X or Y.
- The incoherent character of human probability was notably discussed by Kahneman and Tversky.

- Predd et al. use a method called the **Coherent Approximation Principle**, which is an aggregation method proposed by Osherson and Vardi (2006).
- CAP proposes to use a least-squares method to find a coherent forecast which is closest to all the individual forecast.
- The Predd project is really neither statistical nor philosophical, but algorithmic. Current research suggests that the current algorithm for solving an arbitrary CAP problem is NP-hard.
- Predd et al. propose another algorithm with faster convergence (I won't cover the algorithm here) but want to say how CAP works

Coherent Approximation Principle

- Let Ω be a finite **outcome space**, subsets of which are called events.
- Consider a **multiset** (or a "bag") of these events $\mathcal{E} = \{E_1 \dots E_m\}$.
- A function $f : \mathcal{E} \mapsto [0, 1]$ is a **forecast** which reflects how confident a judge is in the events in $\mathcal{E}$.
- For any **forecast** function f, there is a corresponding vector $\mathbf{f}$, $(f(E_1), f(E_2) \dots f(E_m)) \in [0, 1]^m$.

Coherent Approximation Principle (cont.)

- Suppose now that there is a **panel** of k judges.
- Let judge i provide a forecast $f_i : \mathcal{E}_i \mapsto \mathbb{R}$. (Since this is $\mathbb{R}$, this of course means probabilities do not have to follow unit measure.) I'm unclear whether Predd just means $\mathbb{R}^+$.
- And let $\mathcal{E}'$ be the mutliset from pooling $\{\mathcal{E}_i\}_{i=1}^{m}$.
- Here, then the **aggregation problem** is simple to find an effective method for constructing $F : \mathcal{E}' \mapsto [0, 1]$, where F is coherent.

- In 2006, it was proposed to reduce this to an optimization problem using least squares for finding an F which is **coherent**.

$$\min \sum_{i=1}^{m} \sum_{E \in \mathcal{E}_i} (F(E) - f_i(E))^2 \tag{1}$$

Coherent Approximation Principle (cont.)

- The rest of Predd's work focuses on constructing a scalable algorithm for effectively finding such an F, but no philosophical defense is given why the least squares method is **normatively important**
- The least squares fit equally weights all forecasts but what if we want to weight an expert forecast more than another?
- Does this notion of aggregation make the collective opinion more **objective**? What is the epistemic status of the aggregated result?

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